Introduction to Post-Tonal Theory

second edition

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Contents

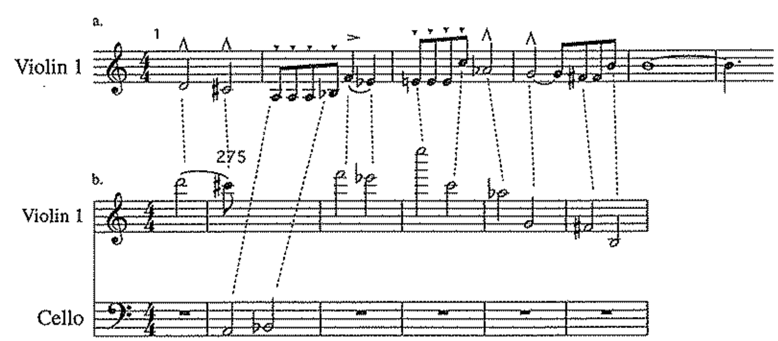
Chapter 1

Basic Concepts and Definitions

Octave Equivalence

There is something special about the octave. Pitches separated by one or more octaves are usually perceived as in some sense *equivalent*. Our musical notation reflects that equivalence by giving the same name to octave-related pitches. The name A, for example, is given not only to some particular pitch, like the A a minor third below middle C, but also to all the other pitches one or more octaves above or below it. Octave-related pitches are called by the same name because they sound so much alike and because Western music treats them as functionally equivalent.

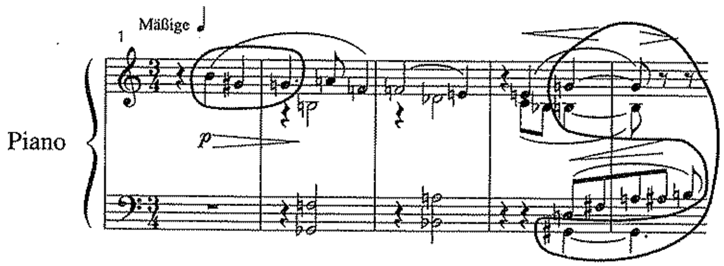
Equivalence is not the same thing as identity. Example 1–1 shows a melody from Schoenberg's String Quartet No. 4, first as it occurs at the beginning of the first movement and then as it occurs a few measures from the end.



Example 1–1 Two equivalent melodies (Schoenberg, String Quartet No. 4).

The two versions are different in many ways, particularly in their rhythm and range. The range of the second version is so wide that the first violin cannot reach all of the notes; the cello has to step in to help. At the same time, however, it is easy to recognize that they are basically the same melody—in other words, that they are equivalent. They are not identical, but they have something basic in common. That something is expressed precisely in the concept of octave equivalence.

In Example 1–2, the opening of Schoenberg's Piano Piece, Op. 11, No. 1, compare the first three notes of the melody with the sustained notes in measures 4–5.



Example 1–2 Two equivalent musical ideas (Schoenberg, Piano Piece, Op. 11, No. 1).

There are many differences between the two collections of notes (register, articulation, rhythm, etc.), but a basic equivalence also. They are equivalent because they both contain a B, a G♯, and a G. When we assume octave equivalence, and other kinds of equivalences we will discuss later, our object is not to smooth out or dismiss the variety of the musical surface. Rather, we seek to discover the relationships that underlie the surface and lend unity and coherence to musical works.

Pitch Class

We will need to distinguish between a *pitch* (a tone with a certain frequency) and a *pitch class* (a group of pitches with the same name). Pitch-class A, for example, contains all the pitches named A. To put it the other way around, any pitch named A is a member of pitch-class A. Sometimes we will speak about specific pitches; at other times we will talk, more abstractly, about pitch classes. When we say that the lowest note on the cello is a C, we are referring to a specific pitch. We can notate that pitch on the second ledger line beneath the bass staff. When we say that the tonic of Beethoven's Fifth Symphony is C, we are referring not to some particular pitch C, but to *pitch-class* C. Pitch-class C is an abstraction and cannot be adequately notated on musical staves. Sometimes, for convenience, we will represent a pitch class using musical notation. In reality, however, a pitch class is not a single thing; it is a class of things, of pitches one or more octaves apart.

Enharmonic Equivalence

In a common-practice tonal music, a B♭ is not the same as an A♯. Even on an equal-tempered instrument like the piano, the tonal system gives B♭ and A♯ different functions and different meanings. For one thing, they represent different scale-degrees. In G major, for example, A♯ is $\sharp\hat{2}$ whereas B♭ is $\flat\hat{3}$, and scale-degrees $\hat{2}$ and $\hat{3}$ have very different musical roles both melodically and harmonically. These distinctions are largely abandoned in post-tonal music, however, where notes that are enharmonically equivalent (like B♭ and A♯) are also functionally equivalent. There may be isolated moments where a composer notates a pitch in what seems like a functional way (sharps for ascending motion and flats for descending, for example). For the most part, however, the notation is functionally arbitrary. It is determined primarily by simple convenience and legibility. The melodies in Example 1–3 are enharmonically equivalent (although the first one is much easier to read).

Example 1–3 Enharmonic equivalence.

Integer Notation

Octave equivalence and enharmonic equivalence leave us with only twelve different pitch classes. All the B♯s, C♮s, and D𝄫s are members of a single pitch class, as are all the C♯s and D♭s, all the C𝄪s, Ds, and E𝄫s, and so on. Composers in the twentieth century have generally continued to use traditional staff notation, where A♭ is notated differently from G♯. However, for our theoretical and analytical purposes, we will also use integers from 0 through 11 to refer to the different pitch classes. Figure 1–1 shows the twelve different pitch classes and some of the contents of each.

integer name	*pitch-class content*
0	B♯, C, D𝄫
1	C♯, D♭
2	C𝄪, D, E𝄫
3	D♯, E♭
4	D𝄪, E, F♭
5	E♯, F, G𝄫
6	F♯, G♭
7	F𝄪, G, A𝄫
8	G♯, A♭
9	G𝄪, A, B𝄫
10	A♯, B♭
11	A𝄪, B, C♭

Figure 1–1

We will use a "fixed *do*" notation: The pitch class containing the Cs is arbitrarily assigned the integer 0 and the rest follows from there.

We didn't have to use integers—we could have assigned arbitrary names to each pitch class—but integers are simple to grasp and to manipulate. They are traditional in music (figured-bass numbers, for example) and useful for representing certain musical relationships. We will never do things to the integers that don't have musical significance. We won't divide integers, because, while dividing 7 into 11 makes numerical sense, dividing G into B doesn't make much musical sense. Other arithmetical operations, however, will prove musically useful. We will, for example, subtract numbers, because, as we will see, subtraction gives us a simple way of talking about intervals. Computing the distance between 7 and 11 by subtracting 7 from 11 makes numerical sense, and the idea of computing the distance between G and B makes musical sense. We will use numbers and arithmetic to model interesting aspects of the music we study. The music itself is not "mathematical" any more than our lives are "mathematical" just because we count our ages in integers. In this book, we will identify pitch classes with either traditional letter notation or integers, whichever seems clearest and easiest in a particular context.

Mod 12

Every pitch belongs to one of the twelve pitch classes. Going up an octave (adding twelve semitones) or going down an octave (subtracting twelve semitones) will just produce another member of the same pitch class. For example, if we start on the E♭ above middle C (a member of pitch class 3) and go up twelve semitones, we end up back on pitch class 3. In other words, in the world of pitch classes, $3 + 12 = 15 = 3$. More generally, any number larger than 11 or smaller than 0 is equivalent to some integer from 0 to 11 inclusive. To figure out which one, just add or subtract 12 (or any multiple of 12). Twelve is called the *modulus,* and our theoretical system frequently will rely upon arithmetic *modulo 12,* for which *mod 12* is an abbreviation. In a mod 12 system, $-12 = 0 = 12 = 24$, and so on. Similarly, −13, −1, 23, and 35 are all equivalent to 11 (and to each other) because they are related to 11 (and to each other) by adding or subtracting 12.

It is easiest to understand these (and other) mod 12 relationships by envisioning a circular clockface, like the one in Figure 1–2.

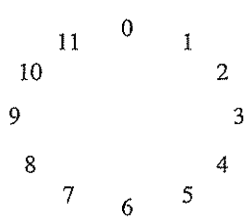


Figure 1–2

In a mod 12 system, moving 12 (or a multiple of 12) in either direction only brings you back to your starting point. As a result, we will generally be dealing only with integers between 0 and 11 inclusive. When we are confronted with a number larger than 11 or smaller than 0, we will usually write it, by adding or subtracting 12, as an integer between 0 and 11. We will sometimes use negative numbers (for example, when we want to suggest the idea of descending), and we will sometimes use numbers larger than 11 (for example, when discussing the distance between two widely separated pitches), but in general we will discuss such numbers in terms of their mod 12 equivalents.

We locate pitches in an extended *pitch space,* ranging in equal-tempered semitones from the lowest to the highest audible tone. We locate pitch classes in a modular *pitch-class space,* as in Example 1–2, which circles back on itself and contains only the twelve pitch classes. It's like the hours of the day or the days of the week. As our lives unfold in time, each hour and each day are uniquely located in linear time, never to be repeated. But we can be sure that, if it's eleven o'clock now, it will be eleven o'clock again in twelve hours (that's a mod 12 system), and that if it's Friday today, it will be Friday again in seven days (that's a mod 7 system). Just as our lives unfold simultaneously in linear and modular time, music unfolds simultaneously in pitch and pitch-class space.

traditional name	*no. of semitones*
unison	0
minor 2nd	1
major 2nd, diminished 3rd	2
minor 3rd, augmented 2nd	3
major 3rd, diminished 4th	4
augmented 3rd, perfect 4th	5
augmented 4th, diminished 5th	6
perfect 5th, diminished 6th	7
augmented 5th, minor 6th	8
major 6th, diminished 7th	9
augmented 6th, minor 7th	10
major 7th	11
octave	12
minor 9th	13
major 9th	14
minor 10th	15
major 10th	16

Figure 1–3

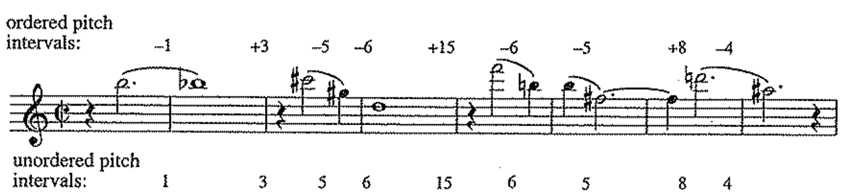
Intervals

Because of enharmonic equivalence, we will no longer need different names for intervals with the same absolute size—for example, diminished fourths and major thirds. In tonal music, such distinctions are crucial; intervals are defined and named according to their tonal function. A third, for example, is an interval that spans three steps of the diatonic scale, while a fourth spans four steps. A major third is consonant, while a diminished fourth is dissonant. In music that doesn't use diatonic scales and doesn't systematically distinguish between consonance and dissonance, it seems cumbersome and even misleading to use traditional interval names. It will be easier and more accurate musically just to name intervals according to the number of semitones they contain. The intervals between C and E and between C and F♭ both contain four semitones and are both instances of interval 4, as are B♯–F♭, C–D𝄪, and so on. Figure 1–3 gives some traditional interval names and the number of semitones they contain.

Pitch Intervals

A pitch interval is simply the distance between two pitches, measured by the number of semitones between them. A pitch interval, which will be abbreviated *ip,* is created when we move from pitch to pitch in pitch space. It can be as large as the range of our hearing or as small as a semitone. Sometimes we will be concerned about the direction of the interval, whether ascending or descending. In that case, the number will be preceded by either a plus sign (to indicate an ascending interval) or a minus sign (to indicate a descending interval). Intervals with a plus or minus sign are called *directed* or *ordered intervals.* At other times, we will be concerned only with the absolute space between two pitches. For such *unordered intervals,* we will just provide the number of semitones between the pitches.

Whether we consider the interval ordered or unordered depends on our particular analytical interests at the time. Example 1–4 shows the opening melody from Schoenberg's String Quartet No. 3, and identifies both its ordered and unordered pitch intervals.



Example 1–4 Ordered and unordered pitch intervals (Schoenberg, String Quartet No. 3).

The ordered pitch intervals focus attention on the contour of the line, its balance of rising and falling motion. The unordered pitch intervals ignore contour and concentrate entirely on the spaces between the pitches.

Ordered Pitch-Class Intervals

A pitch-class interval is the distance between two pitch classes. A pitch-class interval, which will be abbreviated *i,* is created when we move from pitch class to pitch class in modular pitch-class space. It can never be larger than eleven semitones. As with pitch intervals, we will sometimes be concerned with ordered intervals and sometimes with unordered intervals. To calculate pitch-class intervals, it is best to think again of a circular clockface. We will consider clockwise movement to be equivalent to movement upward, and counterclockwise movement equivalent to movement downward. With this in mind, the ordered interval from C♯ to A, for example, is –4 or +8. In other words, from pitch-class C♯, one can go either up eight semitones or down four semitones to get to pitch-class A. This is because +8 and –4 are equivalent (mod 12). It would be equally accurate to call that interval 8 or –4. By convention, however, we will usually denote ordered pitch-class intervals by an integer from 0 to 11. To state this as a formula, we can say that the ordered interval from pitch class x to pitch class y is y – x (mod 12). Notice that the ordered pitch class interval from A to C♯ (1 – 9 = –8 (mod 12) = 4) is different from that from C♯ to A (8), since, when discussing ordered pitch-class intervals, order matters. Four and 8 are each other's *complement mod 12,* because they add up to 12, as do 0 and 12, 1 and 11, 2 and 10, 3 and 9, and 5 and 7. Six is its own complement mod 12.

Figure 1–4 calculates some ordered pitch-class intervals using the formula.

The ordered pitch-class interval from C♯ to E♭ is 3 – 1 = 2
from E♭ to C♯ is 1 – 3 = 10
from B to F is 5 – 11 = 6
from D to B♭ is 10 – 2 = 8
from B♭ to C♯ is 1 – 10 = 3

Figure 1–4

You will probably find it faster just to envision a musical staff, keyboard, or a clockface. To find the ordered pitch-class interval between C♯ and A, just envision the C♯ and then count the number of half-steps you will need to go upward (if you are envisioning a staff or keyboard) or clockwise (if you are envisioning a clockface) to the nearest A.

Unordered Pitch-Class Intervals

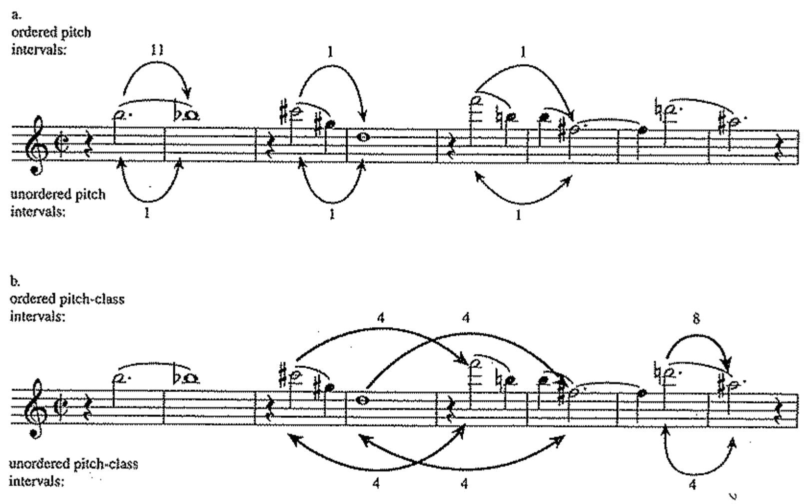
For unordered pitch-class intervals, it no longer matters whether you count upward or downward. All we care about is the space between two pitch classes. Just count from one pitch class to the other by the shortest available route, either up or down. The formula for an unordered pitch-class interval is x – y (mod 12) *or* y – x (mod 12), whichever is smaller. The unordered pitch-class interval between C♯ and A is 4, because 4 (1 – 9 = –8 = 4) is smaller than 8 (9 – 1 = 8). Notice that the unordered pitch-class interval between C♯ and A is the same as that between A and C♯. It is 4 in both cases, since from A to the nearest C♯ is 4 and from C♯ to the nearest A also is 4. Including the unison, 0, there are only seven different unordered pitch-class intervals, because, to get from one pitch class to any other, one never has to travel farther than six semitones. Figure 1–5 calculates some unordered pitch-class intervals using the formula. The correct answer is underlined.

The unordered pitch-class interval between	C♯ and E♭	is 3 − 1 = 2	or	1 − 3 = 10
	E♭ and C♯	is 1 − 3 = 10	or	3 − 1 = 2
	B and F	is 5 − 11 = 6	or	11 − 5 = 6
	D and B♭	is 10 − 2 = 8	or	2 − 10 = 4
	B♭ and C♯	is 1 − 10 = 3	or	10 − 1 = 9

Figure 1–5

Again, you will probably find it faster just to envision a clockface, musical staff, or keyboard. To find the unordered pitch-class interval between B♭ and F♯, for example, just envision a B♭ and count the number of semitones to the nearest available F♯ (4).

In Example 1–5a (again the opening melody from Schoenberg's String Quartet No. 3), the first interval is ordered pitch-class interval 11, to be abbreviated as i11.



Example 1–5 Ordered and unordered pitch-class intervals (Schoenberg, String Quartet No. 3).

That's because to move from B to B♭ one moves −1 or its mod 12 equivalent, i11. Eleven is the name for descending semitones or ascending major sevenths or their compounds. If the B♭ had come before the B, the interval would have been i1, which is the name for ascending semitones or descending major sevenths or their com-

pounds. And that is the interval described by the two subsequent melodic gestures, C♯–D and F–F♯. As ordered pitch-class intervals, the first is different from the second and third. As unordered pitch-class intervals, all three are equivalent. In Example 1–5b, two statements of i4 are balanced by a concluding i8; all three represent unordered pitch-class interval 4.

Interval Class

An unordered pitch-class interval is also called an *interval class*. Just as each pitch-class contains many individual pitches, so each interval class contains many individual pitch intervals. Because of octave equivalence, compound intervals—intervals larger than an octave—are considered equivalent to their counterparts within the octave. Furthermore, pitch-class intervals larger than six are considered equivalent to their complements in mod 12 (0 = 12, 1 = 11, 2 = 10, 3 = 9, 4 = 8, 5 = 7, 6 = 6). Thus, for example, intervals 23, 13, 11, and 1 are all members of interval class 1. Figure 1–6 shows the seven different interval classes and some of the contents of each.

interval class	0	1	2	3	4	5	6
pitch intervals	0,12,24	1,11,13	2,10,14	3,9,15	4,8,16	5,7,17	6,18

Figure 1–6

We thus have four different ways of talking about intervals: ordered pitch interval, unordered pitch interval, ordered pitch-class interval, and unordered pitch-class interval. If in some piece we come across the musical figure shown in Example 1–6, we can describe it in four different ways.

ordered pitch interval: +19
unordered pitch interval: 19
ordered pitch-class interval: 7
unordered pitch-class interval: 5

Example 1–6 Four ways of describing an interval.

If we call it a +19, we have described it very specifically, conveying both the size of the interval and its direction. If we call it a 19, we express only its size. If we call it a 7, we have reduced a compound interval to its within-octave equivalent. If we call it a 5, we have expressed the interval in its simplest, most abstract way. None of these labels is better or more right than the others—it's just that some are more concrete and specific while others are more general and abstract. Which one we use will depend on what musical relationship we are trying to describe.

It's like describing any object in the world—what you see depends upon where you stand. If you stand a few inches away from a painting, for example, you may be aware of the subtlest details, right down to the individual brushstrokes. If you stand back a bit, you will be better able to see the larger shapes and the overall design. There is no single "right" place to stand. To appreciate the painting fully, you have to be willing to move from place to place. One of the specially nice things about music is that you can hear a single object like an interval in many different ways at once. Our different ways of talking about intervals will give us the flexibility to describe many different kinds of musical relationships.

Interval-Class Content

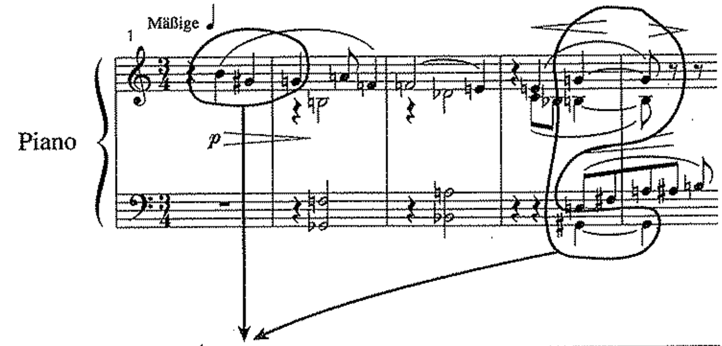
The quality of a sonority can be roughly summarized by listing all the intervals it contains. To keep things simple, we will generally take into account only interval classes (unordered pitch-class intervals). The number of interval classes a sonority contains depends on the number of distinct pitch classes in the sonority. The more pitch classes, the greater the number of interval classes. Figure 1–7 summarizes the number of interval classes in sonorities of all sizes. (We won't bother including the occurrences of interval class 0, which will always be equal to the number of pitch classes in the sonority.)

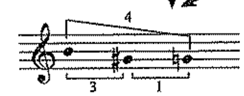
no. of pitch classes	*no. of interval classes*
1	0
2	1
3	3
4	6
5	10
6	15
7	21
8	28
9	36
10	45
11	55
12	66

Figure 1–7

For any given sonority, we can summarize the interval content in scoreboard fashion by indicating, in the appropriate column, the number of occurrences of each of the six interval classes, again leaving out the occurrences of interval class 0. Such a scoreboard conveys the essential sound of a sonority. Notice that now we are counting all of the intervals in the sonority, not just those formed by notes that are right next to each other. That is because all of the intervals contribute to the overall sound.

Example 1–7 refers to the same passage and the same three-note sonority discussed back in Example 1–2.

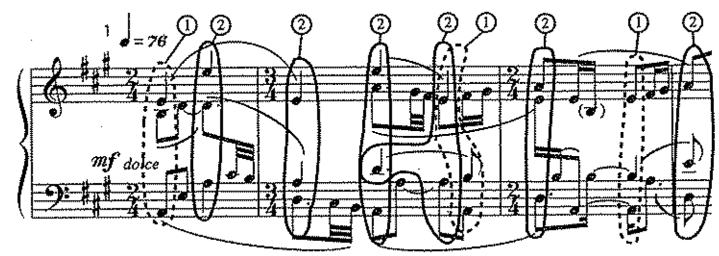


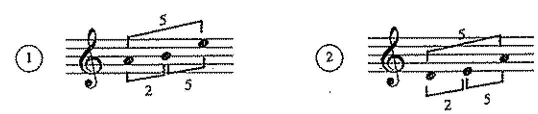


interval class	1	2	3	4	5	6
no. of occurrences	1	0	1	1	0	0

Example 1–7 Interval-class content of a three-note motive (Schoenberg, Piano Piece, Op. 11, No. 1).

Like any three-note sonority, it contains three intervals, in this case one occurrence each of interval classes 1, 3, and 4 (no 2s, 5s, or 6s). How different this is from the sonorities preferred by Stravinsky in the passage from his opera *The Rake's Progress,* shown in Example 1–8! These contain only 2s and 5s.





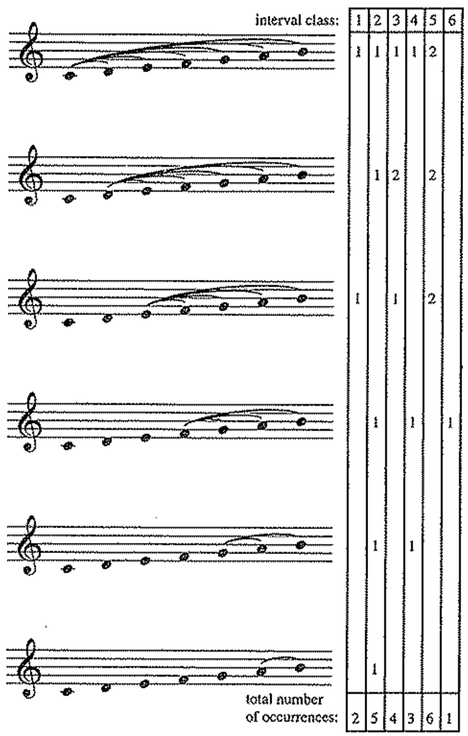
interval class	1	2	3	4	5	6
no. of occurrences	0	1	0	0	2	0

Example 1–8 Interval-class content of a contrasting three-note motive (Stravinsky, *The Rake's Progress,* Act I).

The difference in sound is clearly suggested by listing the interval-class content of the sonorities. The interval-class content is usually presented as a string of six numbers with no spaces intervening. This is called an *interval vector.* The first number in an interval vector gives the number of occurrences of interval class 1; the second gives the number of occurrences of interval class 2; and so on. The interval vector for the sonority in Example 1–7 is 101100, and the interval vector for the sonority in Example 1–8 is 010020.

We can construct a vector like this for sonorities of any size or shape. A tool like the interval vector would not be nearly so necessary for talking about traditional tonal music. There, only a few basic sonorities—four kinds of triads and five kinds of seventh chords—are regularly in use. In post-tonal music, however, we will confront a huge variety of musical ideas. The interval vector will give us a convenient way of summarizing their basic sound.

Even though the interval vector is not as necessary a tool for tonal music as for post-tonal music, it can offer an interesting perspective on traditional formations. Example 1–9 calculates the interval vector for the major scale.



Example 1–9 Interval vector for the major scale.

Notice our methodical process of extracting each interval class. First, the intervals formed with the first note are extracted, then those formed with the second note, and so on. This ensures that we find all the intervals and don't overlook any. As with any seven-note collection, there are 21 intervals in all.

Certain intervallic properties of the major scale are immediately apparent from the interval vector. It has only one tritone (fewer than any other interval) and six occurrences of interval-class 5, which contains the perfect fourth and fifth (more than any other interval). This probably only confirms what we already knew about this scale, but the interval vector makes the same kind of information available about less familiar collections. The interval vector of the major scale has another interesting property—it contains a different number of occurrences of each of the interval classes. This is an extremely important and rare property (only three other collections have it) and it is one to which we will return. For now, the important thing is the idea of describing a sonority in terms of its interval-class content.

BIBLIOGRAPHY

The material presented in Chapter 1 (and in Chapters 2 and 3 as well) is also discussed in three widely used books: Allen Forte, *The Structure of Atonal Music* (New Haven: Yale University Press, 1973); John Rahn, *Basic Atonal Theory* (New York: Longman, 1980); and George Perle, *Serial Composition and Atonality,* 6th ed., rev. (Berkeley and Los Angeles: University of California Press, 1991). Two important books offer profound new perspectives on this basic material, and much else besides: David Lewin, *Generalized Musical Intervals and Transformations* (New Haven: Yale University Press, 1987); and Robert Morris, *Composition with Pitch Classes* (New Haven: Yale University Press, 1987). See also Michael Friedmann's brilliant *Ear Training for Twentieth-Century Music* (New Haven: Yale University Press, 1990).

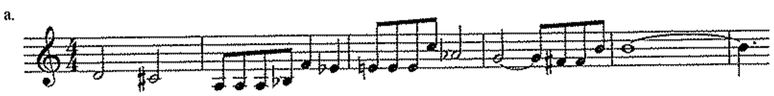
Exercises

THEORY

I. Integer Notation: Any pitch can be represented by an integer. In the commonly used "fixed *do*" notation, C = 0, C♯ = 1, D = 2, and so on.

1. Represent the following melodies as strings of integers:

a.



b.

c.

d.

2. Show at least two ways each of the following strings of integers can be notated on a musical staff:
   a. 0 1 3 9 2 11 4 10 7 8 5 6
   b. 2 4 1 2 4 6 7 6 4 2 4 2 1 2
   c. 0 11 7 8 3 1 2 10 6 5 4 9
   d. 11 8 7 9 5 4

II. Pitch Class and Mod 12: Pitches that are one or more octaves apart are equivalent members of a single pitch class. Because an octave contains twelve semitones, pitch classes can be discussed using arithmetic modulo 12 (mod 12), in which any integer larger than 11 or smaller than 0 can be reduced to an integer from 0 to 11 inclusive.

1. Using mod 12 arithmetic, reduce each of the following integers to an integer from 0 to 11 inclusive:
   a. 15
   b. 27
   c. 49
   d. 13
   e. –3
   f. –10
   g. –15

2. List at least three integers that are equivalent (mod 12) to each of the following integers:
   a. 5
   b. 7
   c. 11

3. Perform the following additions (mod 12):
   a. 6 + 6
   b. 9 + 10
   c. 4 + 9
   d. 7 + 8

4. Perform the following subtractions (mod 12):
   a. 9 – 10
   b. 7 – 11
   c. 2 – 10
   d. 3 – 8

III. Intervals: Intervals are identified by the number of semitones they contain.

1. For each of the following traditional interval names, give the number of semitones in the interval:
   a. major third
   b. perfect fifth
   c. augmented sixth
   d. diminished seventh
   e. minor ninth
   f. major tenth

2. For each of the following numbers of semitones, give at least one traditional interval name:
   a. 4
   b. 6
   c. 9
   d. 11
   e. 15
   f. 24

IV. Ordered Pitch Intervals: A pitch interval is the interval between two pitches, counted in semitones. + indicates an ascending interval; – indicates a descending interval.

1. Construct the following ordered pitch intervals on a musical staff, using middle C as your starting point.
   a. +15
   b. –7
   c. –4
   d. +23

2. For the following melodies, identify the ordered pitch interval formed by each pair of adjacent notes.

a.

b.

c.

d.

V. Unordered Pitch Intervals: An unordered pitch interval is simply the space between two pitches, without regard to the order (ascending or descending) of the pitches.

1. Construct the following unordered pitch intervals on a musical staff, using middle C as the lowest note.
a. 15
b. 4
c. 7
d. 11
e. 23

2. For the melodies in Exercise IV/2, identify the unordered pitch interval formed by each pair of adjacent notes.

VI. Ordered Pitch-Class Intervals: A pitch-class interval is the interval between two pitch classes. On the pitch-class clockface, always count clockwise from the first pitch class to the second.

1. For each of the melodies in Exercise IV/2, identify the ordered pitch-class interval formed by each pair of adjacent notes.

2. Which ordered pitch-class intervals are formed by the following ordered pitch intervals?
a. +7
b. −7
c. +11
d. +13
e. −1
f. −6

VII. Unordered Pitch-Class Intervals: An unordered pitch-class interval is the shortest distance between two pitch classes, regardless of the order in which they occur. To calculate an unordered pitch-class interval, take the shortest route from the first pitch class to the second, going either clockwise or counterclockwise on the pitch-class clockface.

1. For each of the melodies in Exercise IV/2, identify the unordered pitch-class interval formed by each pair of adjacent notes.

2. An unordered pitch-class interval is also called an interval class. Give at least three pitch intervals belonging to each of the six interval classes.

VIII. Interval Vector: Any sonority can be classified by the intervals it contains. The interval content is usually shown as a string of six numbers called an interval vector. The first number in the interval vector gives the number of occurrences of interval class 1; the second number gives the number of occurrences of interval class 2; and so on.

1. For each of the following collections of notes, give the interval-class content, expressed as an interval vector.
a. 0, 1, 3, 4, 6, 7, 9, 10
b. 0, 2, 4, 6, 8, 10
c. 2, 3, 7
d. the augmented triad
e. the pentatonic scale
f. 1, 5, 8, 9

2. For each of the following interval vectors, try to construct the collection that it represents.
a. 111000
b. 004002
c. 111111
d. 303630

ANALYSIS

I. Webern, Symphony Op. 21, Thema, mm. 1–11, clarinet melody: How is this melody organized? What patterns of recurrence do you notice? Begin by identifying all of the ordered and unordered pitch and pitch-class intervals. (*Hint:* Consider not only the intervals formed between adjacent notes of the melody, but also the intervals that frame it, for example, the interval between the first note and the last, between the second note and the second-to-last, and so on.)

II. Schoenberg, Piano Concerto, mm. 1–8, right-hand melody: Are there any intervals or motives that recur? (*Hint:* The melody is framed by its first note, E♭, which also is its highest note, its lowest note, A♭, and its final note, G. Are there varied repetitions of this three-note motive directly within the melody?)

III. Stravinsky, "Musick to heare" from *Three Shakespeare Songs,* mm. 1–8, flute melody: What patterns of intervallic recurrence do you see? (*Hint:* Think of the first four notes as a basic motivic/intervallic structure.)

IV. Crawford, *Diaphonic Suite* No. 1 for Oboe or Flute, mm. 1–18: How is this melody organized? The composer thought of this melody as a kind of musical poem and indicated the lines of the poem with double bars (at the end of mm. 5, 9, 14, and 18). Describe the musical "rhymes" and any other intervallic or motivic recurrences. (*Hint:* Take the first three notes, ip +2 followed by ip −1, as a basic motive.)

V. Varese, *Octandre,* mm. 1–5, oboe melody: How is this melody organized? (*Hint:* Consider both the first four notes, G♭–F–E–D♯, and the highest three of those, G♭–E–D♯, as basic motivic units.)

VI. Babbitt, "The Widow's Lament in Springtime," mm. 1–6, vocal melody: How is this melody organized intervallically and motivically? How many different ordered pitch-class intervals are used? Amid this variety, what are the sources of unity? (*Hint:* Consider the framing intervals—first to last, second to second-to-last, etc., as well as the direct melodic intervals.)

EAR-TRAINING AND MUSICIANSHIP

I. Webern, Symphony Op. 21, Thema: Sing the clarinet melody, accurately and in tempo, using pitch-class integers in place of the traditional solfege syllables. To maintain a single syllable for each note, sing "oh" for 0, "sev" for 7, and "lev" for 11.

II. Schoenberg, Piano Concerto, mm. 1–8, right-hand melody: Sing the melody, accurately and in tempo, using pitch-class integers.

III. Stravinsky, "Musick to heare" from *Three Shakespeare Songs,* mm. 1–8, flute melody: Sing the melody, accurately and in tempo, using pitch-class integers.

IV. Crawford, *Diaphonic Suite* No. 1 for Oboe or Flute, mm. 1–18: Play this melody, accurately and in tempo, on any appropriate instrument.

V. Varese, *Octandre,* mm. 1–5, oboe melody: Play this melody, accurately and in tempo, on any appropriate instrument.

VI. Babbitt, "The Widow's Lament in Springtime," mm. 1–6, vocal melody: Sing the melody, accurately and in tempo, using either the words of the text (by William Carlos Williams) or pitch-class integers.

VII. Identify melodic and harmonic intervals played by your instructor as ordered and unordered pitch and pitch-class intervals.

VIII. From a given note, learn to sing above or below by a specified pitch interval (within the constraints of your vocal range).

COMPOSITION

I. Write two short melodies of contrasting character for solo flute or oboe that make extensive use of one of the following motives: ip<+3, −11>, ip<+3, −4>, i<8, 2, 1>, or i<2, 11>.

II. Write brief duets for soprano and alto that have the following characteristics:

1. Begin with middle C in the alto and the B eleven semitones above it in the soprano.
2. Use whole notes only, as in first-species counterpoint.
3. The interval between the parts must be a member of ic1, ic2, or ic6.
4. Each part may move up or down only by ip1, ip2, ip3 or ip4.
5. End on the notes you began with.
6. Try to give an attractive, purposeful shape to both melodies.

Analysis 1

Webern, "Wie bin ich froh!" from Three Songs, Op. 25
Schoenberg, "Nacht," from *Pierrot Lunaire,* Op. 21

Listen several times to a recording of "Wie bin ich froh!"—a song written by Anton Webern in 1935. We will concentrate on the first five measures, shown in Example A1–1.

Example A1–1 Webern, "Wie bin ich froh!" from Three Songs, Op. 25 (mm. 1–5).

Here is a translation of the first part of the text, a poem by Hildegarde Jone.

Wie bin ich froh!	How happy I am!
noch einmal wird mir alles grün	Once more all grows green around me
und leuchtet so!	And shines so!

The music may sound at first like disconnected blips of pitch and timbre. A texture that sounds fragmented, that shimmers with hard, bright colors, is typical of Webern. Such a texture is sometimes called "pointillistic," after the technique of painting with sharply defined dots or points of paint. Gradually, with familiarity and with some knowledge of pitch and pitch-class intervals, the sense of each musical fragment and the interrelations among the fragments will come into focus.

The lack of a steady meter may initially contribute to the listener's disorientation. The notated meters, 3/4 and 4/4, are hard to discern by ear, since there is no regular pattern of strong and weak beats. The shifting tempo—there are three ritards in this short passage—confuses matters further. The music ebbs and flows rhythmically rather than following some strict pattern. Instead of searching for a regular meter, which certainly does not exist here, let's focus instead on the smaller rhythmic figures in the piano part, and the ways they group to form larger rhythmic shapes.

The piano part begins with a rhythmic gesture consisting of three brief figures: a sixteenth-note triplet, a pair of eighth-notes, and a four-note chord. Except for two isolated single tones, the entire piano part uses only these three rhythmic figures. But, except for measure 2, the three figures never again occur in the same order or with the same amount of space between them. The subsequent music pulls apart, plays with, and reassembles the opening figures. Consider the placement of the sixteenth-note triplet, which becomes progressively more isolated as the passage progresses. In the pickup to measure 1 and in measure 2, it is followed immediately by the pair of eighth-notes. In measure 3, it is followed immediately, not by a pair of eighth-notes, but by a single note. At the beginning of measure 4, it is again followed by a single note, but only after an eighth-note rest. At the end of measure 4, it is even more isolated—it is followed by five eighth-notes' worth of rests. The shifting placement of the rhythmic figures gives a gently syncopated feeling to the piano part. You can sense this best if you play the piano part or tap out its rhythms.

Now let's turn to the melodic line. Begin by learning to sing it smoothly and accurately. This is made more difficult by the wide skips and disjunct contour so typical of Webern's melodic lines. Singing the line will become easier once its organization is better understood. Using the concepts of pitch and pitch class, and of pitch and pitch-class intervals, we can begin to understand how the melody is put together.

There is no way of knowing, in advance, which intervals or groups of intervals will turn out to be important in organizing this, or any, post-tonal work. Each of the post-tonal pieces discussed in this book tends to create and inhabit its own musical world, with musical content and modes of progression that may be, to a significant extent, independent of other pieces. As a result, each time we approach a new piece, we will have to pull ourselves up by our analytical bootstraps. The process is going to be one of trial and error. We will look, initially, for recurrences (of notes and intervals) and patterns of recurrence. It often works well to start right at the beginning, to see the ways in which the initial musical ideas echo throughout the line.

In "Wie bin ich froh," it turns out that the first three notes, G–E–D♯, and the intervals they describe play a particularly central role in shaping the melody. Let's begin by considering their ordered pitch intervals (see Figure A1–1).

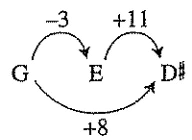
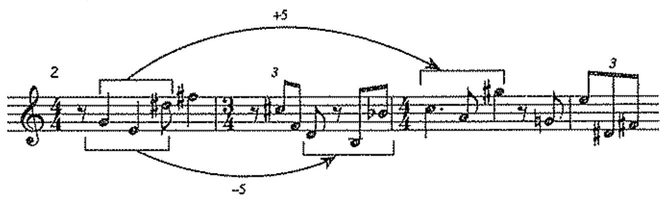


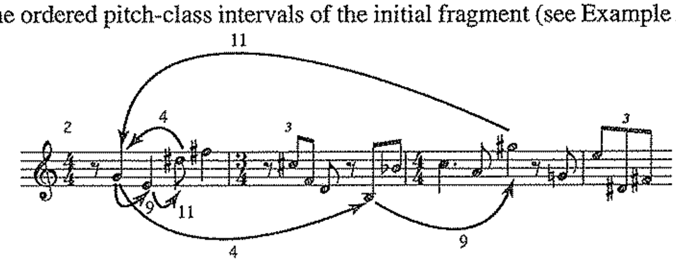
Figure A1–1

The same ordered pitch intervals occur in the voice in two other places, in measure 3 (D–B–B♭) and again in measure 4 (C–A–G♯). (See Example A1–2.)



Example A1–2 Three fragments with the same ordered pitch intervals.

Sing these three fragments, then sing the whole melody and listen to how these fragments help give it shape. The second fragment starts five semitones before the first, while the third fragment starts five semitones higher. That gives a sense of symmetry and balance to the melody, with the initial fragment lying halfway between its two direct repetitions. Furthermore, the second fragment brings in the lowest note of the melody, B, while the third fragment brings in the highest note, G♯. These notes, together with the initial G, create a distinctive frame for the melody as a whole, one which replicates the ordered pitch-class intervals of the initial fragment (see Example A1–3).



Example A1–3 A melodic frame (first note, lowest note, highest note) that replicates the ordered pitch-class intervals of the initial fragment.

Composers of post-tonal music often find ways of projecting a musical idea simultaneously on the musical surface and over larger musical spans. This kind of *composing-out* is an important unifying device and it is one to which we will often return.

The three melody notes at the beginning of measure 3, C♯–F–D, also relate to the opening three-note figure, but in a more subtle way. They use the same pitch intervals as the first three notes of the melody (3, 8, and 11), but the intervals occur in a different order. In addition, two of the three intervals have changed direction (see Figure A1–2).

Figure A1–2

In other words, the fragment C♯–F–D has the same unordered pitch intervals as the opening figure, G–E–D♯. The relationship is not as obvious as the one shown in Example A1–2, but it is still not hard to hear. Sing the two fragments, then sing the entire melody and listen for the resemblance (see Example A1–4).

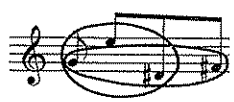
Example A1–4 Two fragments with the same unordered pitch intervals.

The first four pitch classes of the melody are the same, and in the same order, as the last four: G–E–D♯–F♯ (see Example A1–5).

Example A1–5 The first four notes and the last four have the same ordered pitch-class intervals.

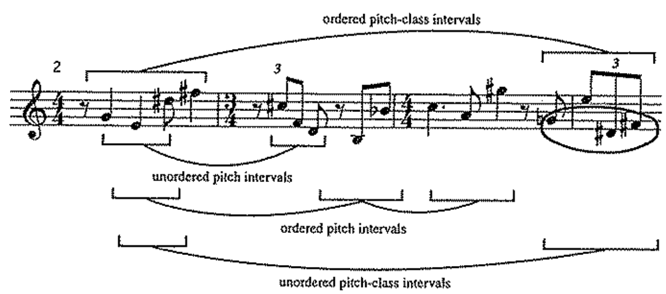
The contours of the two phrases (their successive ordered pitch intervals) are different, but the ordered pitch-class intervals are the same: 9–11–3. This similarity between the beginning and end of the melody is a nice way of rounding off the melodic phrase and of reinforcing the rhyme in the text: "Wie bin ich froh! . . . und leuchtet so!" Sing these two fragments and listen for the intervallic equivalence that lies beneath the change in contour.

By changing the contour the second time around, Webern makes something interesting happen. He puts the E up in a high register, while keeping the G, D♯, and F♯ together in a low register. Consider the unordered pitch-class intervals in that registrally defined three-note collection (G–D♯–F♯). It contains interval classes 1 (G–F♯), 3 (D♯–F♯), and 4 (G–D♯). These are exactly the same as those formed by the first three notes (G–E–D♯) of the figure: E–D♯ is 1, G–E is 3, and G–D♯ is 4 (see Example A1–6).



Example A1–6 A registral grouping and a melodic figure contain the same unordered pitch-class intervals.

The melodic line is thus supercharged with a single basic motive. The entire melody develops musical ideas presented in the opening figure, sometimes by imitating its ordered pitch intervals, sometimes by imitating only its unordered pitch intervals, and sometimes, still more subtly, by imitating its ordered or unordered pitch-class intervals (see Example A1–7).



Example A1–7 Development of the initial melodic figure.

Knowledge of the intervallic structure of the melody should make it easier to hear it clearly and to sing it accurately. Sing the melody again, concentrating on the motivic and intervallic interplay shown in Example A1–7.

The piano accompaniment develops and reinforces the same musical ideas. Rather than trying to deal with every note, let's just concentrate on the sixteenth-note triplet figure that comes five times in the passage. When it occurs in measure 2 (G–E–D♯), it

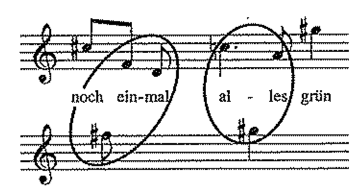
contains the same pitches and thus the same ordered pitch intervals as the beginning of the melody: –3, +11. In measure 3, different pitches are used (C–A–G♯), but the ordered pitch intervals are the same: –3, +11. When it occurs in the pickup to measure 1 (F♯–F–D) and at the end of measure 4 (B–B♭–G), it has the same ordered pitch intervals, but reversed: +11, –3.

The remaining occurrence of the figure, at the beginning of measure 4 (C–A–C♯), is somewhat different from these. Its ordered pitch intervals are –3, +16. It is not comparable to the others in terms of its pitch intervals or even its ordered pitch-class intervals. To understand its relationship to the other figures we will have to consider its interval classes. It contains a 3 (C–A), a 1 (C–C♯), and a 4 (A–C♯). Its interval-class content (the interval vector is 101100) is thus the same as the first three notes of the voice melody (see Example A1–8).

Example A1–8 Accompanimental figures derived from the initial melodic idea.

In fact, all of the three-note figures we have discussed in both the vocal and piano parts have this same interval vector. That is one reason the piece sounds so unified. Play each of the three-note figures in the piano part and listen for the ways they echo the beginning of the voice part—sometimes overtly, sometimes more subtly.

So far, we have talked about the voice part and the piano part separately. But, as in more traditional songs, the piano part both makes sense on its own and accompanies and supports the voice. For a brief example, consider the two single notes in the piano part, the F♯ in measure 3 and the G♯ in measure 4. In both cases, the piano note, together with nearby notes in the voice, creates a three-note collection with that familiar interval-class content: 101100 (see Example A1–9).



Example A1–9 Piano and voice together create collections with interval classes 1, 3, and 4 (interval vector 101100).

The passage, at least as far as we have discussed it, is remarkably unified intervallically. It focuses intensively on the pitch intervals 3, 8, and 11 and, more abstractly, on interval classes 1, 3, and 4. The passage is saturated with these intervals and with motivic shapes created from them. Some of the relationships are simple and direct—we can discuss them in terms of shared pitch intervals. Others are subtly concealed and depend on the more abstract concepts of pitch-class interval and interval-class content. With our knowledge of pitch and pitch-class intervals, we can accurately describe a whole range of motivic and intervallic relationships.

The same sort of intensive intervallic concentration is at work in "Nacht," one of the twenty-one short movements that make up Arnold Schoenberg's *Pierrot Lunaire. Pierrot* is one of this century's acknowledged masterpieces and probably Schoenberg's best-known work. Many factors contribute to its stunning effect. The instrumentation is wonderfully varied. The work is scored for a singer and a small instrumental ensemble (piano, flute/piccolo, clarinet/bass clarinet, violin/viola, and cello) in such a way that no two of the twenty-one movements have the same instrumentation.

The singer uses a vocal technique known as *Sprechstimme* (speech-song), a kind of declamation that is halfway between speech and song. The notated pitch should not be sustained but should be slid away from, in the manner of speech. As to whether the notated pitch need be sung accurately in the first place, there is considerable controversy. Some singers lean toward the speech part of speech-song, following only the approximate contours of the notated line; others try to give a clear indication of the actual pitches specified. As we will see, the pitches in the vocal part so consistently reproduce intervals and motives from the instrumental parts that singers should probably touch the notated pitches accurately before sliding away. Listen to a recording of *Pierrot Lunaire,* concentrating on "Nacht," the eighth movement. The score for measures 1–10 is given in Example A1–10, with a translation of the first stanza of the text, itself a German translation of a poem by Albert Giraud.

Finstre, schwarze Riesenfalter	Dark, black giant butterflies
Töteten der Sonne Glanz.	Have obliterated the rays of the sun.
Ein geschlossnes Zauberbuch,	Like an unopened magic-book,
Ruht der Horizont—verschwiegen.	The horizon rests—concealed.

Schoenberg calls this piece a passacaglia. A passacaglia is a continuous variation form that uses a bass ostinato. In this piece, the ostinato consists of the three-note figure E–G–E♭. After the introduction (measures 1–3), this figure occurs once in each measure of this passage. Play this figure as it occurs in each measure, noticing how it moves from voice to voice and register to register. In measures 8 and 9, each tone of the figure is elaborated, in diminution, by a rapid statement of the same figure transposed (see Example A1–11).

Example A1–10 Schoenberg, "Nacht," from *Pierrot Lunaire* (mm. 1–10).

Example A1–11 The head-motive (E–G–E♭) elaborated, in diminution, by transposed versions of itself.

In measure 10, the passage comes to a striking conclusion when the same figure appears in the voice part. This is the only time in the piece that the singer actually *sings.* Her doing so in such a low, dark register and on such musically significant notes adds to the emotional impact of the word *verschwiegen* (concealed), a word that seems to crystallize the ominous, foreboding nature of the entire text.

Let's examine the intervallic makeup of that repeated figure: E–G–E♭. Its ordered pitch intervals are +3, and –4, and (from the first note to the last) –1. These intervals permeate the entire musical fabric. Consider, for example, the tune stated first in the bass clarinet beginning in measure 4, and then imitated in the cello (measure 5), the left hand of the piano (measure 6), and, in part, the right hand of the piano (measure 7) (see Example A1–12).

Example A1–12 The head-motive expanded and developed into a recurring melody.

The tune begins, of course, with E–G–E♭ as its head-motive. It then takes the interval –1, spanned by E–E♭, and extends it into a lengthy chromatic descent. The tune ends with a three-note figure that introduces two new pitch intervals, +9 and +8.

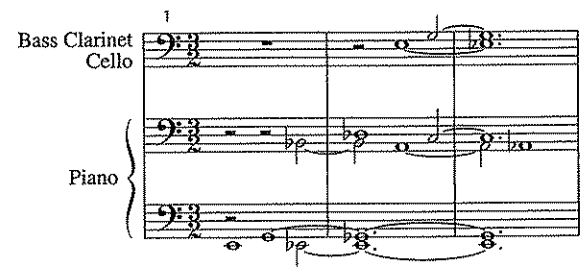
This new figure, B♭–A–G♭, does not have any obvious relationship to the head-motive, E–G–E♭. It has a different contour and different pitch intervals. To understand the relationship, we will have to consider the unordered pitch-class intervals of the two figures. Both have a 1, a 3, and a 4. (Their shared interval vector is thus 101100—coincidentally the same as that of the main motive in Webern's song, discussed earlier.) Find those three intervals in each of the figures. From the perspective of interval class, we can hear the second figure as a development of the first. Sing the tune shown in Example A1–12 and listen for the familiar head-motive, its continuation into a chromatic descent, and its development in the concluding figure.

In light of these observations, it becomes clear how carefully Schoenberg has notated the pitches of the voice part. Consider its first melodic gesture, shown in Example A1–13.

Example A1–13 Motivic penetration of the *Sprechstimme* part.

In its initial chromatic descent from D♭ to A and the leap upward from A to G♭, it exactly traces the last part of the melody shown in Example A1–12. Then, by moving down to F, it tacks on an additional, overlapping version of the three-note figure involving pitch intervals 8 and 9. Surely these pitches should be clearly indicated by the performer! Try it yourself, first trying to indicate the notated pitches and then mainly chanting. Which do you prefer?

The introduction (measures 1–3) not only sets an appropriately gloomy mood with its use of the lowest, darkest possible register, but also introduces the main intervallic material in a subtle way. To make it easier to see and hear what is going on, the music is written an octave higher in Example A1–14.



Example A1–14 Motivic saturation of the introduction.

Of the six distinct musical lines here, all but one descend by semitone from the initial pitch. The melodic interval of –1, of course, anticipates the many chromatic descents that are coming up later in the music. Even more striking, however, are the relationships between the lines. In the lowest register, the first three notes are E–G–E♭, our familiar head-motive. The second note of the motive, the G, is also the first note of a transposed statement of the motive: G–B♭–G♭. The second note of that statement, the B♭, becomes the first note of a new statement: B♭–D♭–A. This process continues upward until the cello and bass clarinet come in with a restatement, an octave higher, of the original E–G–E♭. One additional statement of the motive, A–C–A♭, begins in the middle of the texture on the second beat of measure 2. In all there are six statements of the motive packed into these three measures. The density is extraordinary; the music of the introduction is motivically saturated. Play these measures and listen for each statement of the motive. The music that follows can be heard as an unpacking of material so intensely presented in the introduction.